[This question paper contains 12 printed pages.]

Your Roll No..............

Sr. No. of Question Paper : 4748 **E**

Unique Paper Code : 32357609

Name of the Paper : DSE-3 : Bio-Mathematics

Name of the Course : **B.Sc. (H) Mathematics**

Semester : VI

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. All the **six** questions are compulsory.
3. Attempt any **two** parts from each question.
4. Use of Scientific Calculator is allowed.

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1. (a) A doctor has to prescribe medicine to the patient. The medicine raises the blood plasma concentration of an average adult by 20 mg l^{-1} and takes 6 h to decay in the blood plasma. The maximum

P.T.O.

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permissible limit of concentration of drug in the body is 40 mg l^{-1}. What time gap he will ensure for maintaining a safe decomposition of drug. Find out the minimum concentration if doses are given at this time interval? The concentration of another drug, decreases by 40% in 20 h. Find how long will it take for this drug to fall to 5% of its initial value. (6)

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(b) Observation on animal tumours indicate that theirsizes obey the Gompertz growth law $\frac{ds}{dt} = ks\ln\left(\frac{S}{s}\right)$ rather than the logistic law. Here k and S are positive constants. By putting y = ln(s), prove that $s(t) = Se^{-Ae^{-kt}}$, where $A = \ln\left(\frac{S}{s_0}\right)$, s_0

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being the size at $t = 0$. Discuss the model describing drug concentration and residual concentration at any time nt_0, in which drug decays according to equation $\frac{dc}{dt} = \frac{-c(t)}{\tau}$. When dose is administered regularly at time $t = 0, t_0, 2t_0, 3t_0, \ldots$ with assumption that each dose raises the drug concentration by fixed amount Co. Find the maximum possible concentration and residue as n increases. (6)

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(c) Consider the following chemical reaction, with the rate constant as q :

$$P + X \rightarrow 7Y.$$

If the reactant P is held at a constant concentration p, derive a system of equations for the

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concentrations of X and Y. Suppose the initial concentrations of X and Y are X_0 and Y_0 respectively. Solve the system of equations to obtain X(t) and Y(t). (6)

(d) Show that Zeeman's heartbeat equations have a unique resting state

$$x = x_a,\ b = -(x_a^3 + a\,x_a).$$

Then derive the single differential equation satisfied by muscle fibre of length x. (6)

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2. (a) Discuss the nature of fixed point and give equation of trajectories for the given system

$$\dot{x} = 6x + 12y$$
$$\dot{y} = 3x + y$$ (6½)

(b) (i) Discuss the two equilibrium states during the heart beat cycle and the role of pacemaker in the heartbeat cycle.

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(ii) Discuss the threshold level and firing of axon in nerve impulse transmission. (6½)

(c) Examine the possibility of periodic solutions of

$$c\ddot{x} + (2 + 3ax + 4bx^2)x = 0$$

Where a, b and c are constants, c being positive. (6½)

(d) Describe the epidemic model and show that population return to equilibrium after the small departure from the equilibrium. (6½)

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3. (a) Consider the system

$$\frac{du}{dt} = u\ (1 - u)\ (u - a) - w$$

$$\frac{dw}{dt} = bu - \gamma w.$$

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Where $0 < a < 1$, $b > 0$, $\gamma \geq 0$.

Linearise the above system about (0,0). Further assuming that $u = \alpha e^{\lambda t}$, $w = \beta e^{\gamma t}$ be solutions of the linearised system about (0,0), show that the rest state (0,0) is locally stable. (6)

(b) Sketch the trajectories of the following system

$$\dot{x} = y$$

$$\dot{y} = \frac{1}{2}(1 - x^2) \quad (6)$$

(c) Define

(i) Bifurcation

(ii) Bifurcation Point

Make the sketches for Pitchfork bifurcation,

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Saddle-node bifurcation and Hopf bifurcation.

(6)

(d) For the iteration scheme $x_{n+1} = \mu x_n(1 - x_n)$, $n \geq 1$,

$$x_0 = \lim_{n\to\infty} x_n$$

Show that there are bifurcations at $\mu = 1$ and $\mu = 3$. (6)

4. (a) Find the constraints on a, b and λ assuming it has a unique rest state, taking the solutions to the travelling wave equations in the form $u = \phi(x + ct)$, $w = \psi(x + ct)$ of the following system of equations $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + u(1-u)(u-a) - w$,

$\frac{\partial w}{\partial t} = bu - \gamma w.$ (6½)

(b) What is Flopf bifurcation? Show that Hopf

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bifurcation holds for the following system

$$\dot{x} = -y + x(\mu - x^2 - y^2)$$

$$\dot{y} = x + y(\mu - x^2 - y^2) \qquad (6½)$$

(c) Provide a full phase plane analysis for the mathematical model of heart beat equations given by

$$\varepsilon \frac{dx}{dt} = -\left(x^3 - Tx + b\right), \qquad T > 0$$

$$\frac{db}{dt} = x - x_0.$$

Where x is the muscle fibre length, b is the chemical control, $\varepsilon > 0$ and (x_0, b_0) is a rest state. (6½)

(d) Show that the following system has limit cycle.

$$\frac{du}{dt} = u(1-u)(u-a) - w + I(t),$$

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$$\frac{dw}{dt} = bu - \gamma w,\ 0 < a < 1,\ b > 0,\ \gamma \geq 0. \qquad (6½)$$

5. (a) Write down the steps of Neighbor Joining Algorithm. From the given distance table of four taxa S_1, S_2, S_3 and S_4, compute R_1, R_2, R_3, R_4 and then form a table of values for $M(S_i, S_j)$ for $1 \leq i \neq j \leq 4$.

	S_1	S_2	S_3	S_4
S_1		1.2	0.9	1.7
S_2			1.1	1.9
S_3				1.6

(6½)

(b) If D and d denote the alleles for tall and dwarf plant and if W and w denote the alleles for round

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and wrinkled seed, then create a Punnett square for a DdWw × ddWw cross pea plant and compute the probability of a tall plant with wrinkled seeds. (6½)

(c) Derive the formula for the Jukes-Cantor distance (d_{JC}) given that all the diagonal entries of Jukes Cantor matrix M^t are $\frac{1}{4}+\frac{3}{4}\left(1-\frac{4}{3}\alpha\right)^t$, where a is the mutation rate. Compute the Jukes-Cantor distance $d_{JC}(S_0, S_1)$ to 4 decimal digits, from the following 40 base table : (6½)

$S_1 \backslash S_0$	A	G	C	T
A	7	0	1	1
G	1	9	2	0
C	0	2	7	2
T	1	0	1	6

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(d) Describe when two trees are considered to be topologically similar. Draw all topologically distinct un-rooted bifurcation trees that could describe the relationship between 3 taxa and 4 taxa. (6½)

6. (a) Define phylogenetic tree, bifurcating tree and unrooted tree with examples of each. (6)

(b) Explain Kimura 2-parameter and 3-parameter models along with their corresponding distance formulas. Write the expression of the log-det distance between S_0 and S_1. (6)

(c) In mice, an allele A for agouti- or gray-brown grizzled fur is dominant over the allele a, which determines a non-agouti color. If an Aa×Aa cross produces 4 offsprings, then compute the probabilities that :

(i) No offspring have agouti fur.

(ii) Exactly 3 of 4 offspring have agouti fur.

(6)

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(d) From the given distance table of four sequences S_1, S_2, S_3 and S_4 of DNA, construct a rooted tree showing the relationship between S_1, S_2, S_3 and S_4 by UPGMA

	S_1	S_2	S_3	S_4
S_1		0.45	0.27	0.53
S_2			0.40	0.50
S_3				0.62

(6)

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